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1. Let  $\ell_1, \dots, \ell_k$  be some nonnegative numbers such that  $\ell_1 + \dots + \ell_k = \ell$ . Find the number of weak compositions (in terms of  $\ell$ ,  $k$ , and  $n$ )  $(a_1, \dots, a_k)$  of  $n$  into  $k$  such that  $a_i \geq \ell_i$ .

2. Let  $n$  be a natural number.

(a) Find an explicit formula for  $S(n, n - 2)$ .

(b) Find an explicit formula for  $S(n, 3)$ .

3. How many numbers must be selected from the set  $[6]$  to guarantee that at least one pair of these numbers add up to 7?

4. Show that  $\int_0^{+\infty} x^n e^{-x} dx = n!$  for all  $n \geq 0$ .