Name: \_\_\_\_\_

Pid: \_\_\_\_\_

1. (10 points) Let  $S = \{n \in \mathbb{N} : x^n + y^n = z^n \text{ has an integer solution}\}$ . Prove that the set S is enumeratable.

2. (10 points) Let  $A, B \subseteq \mathbb{N}$  be enumeratable. Show that  $A \cup B$  is also enumeratable.

3. (10 points) We say that a real number  $\alpha$  be computable iff there is a computable function  $a : \mathbb{Q} \to \mathbb{Q}$  such that  $|\alpha - a(\epsilon)| \leq \epsilon$  for any rational  $\epsilon > 0$ .

Show that a number  $\alpha < 1$  is computable iff the function  $f : \mathbb{N} \to \{0, 1, \dots, 9\}$  such that f(i) is the *i*th digit of the base-10 representation of  $\alpha$  is computable.

4. (10 points) Let  $X, A, B \subseteq \mathbb{N}$  such that  $X = A \Delta B$  (X is the symmetric difference of A and B) and A and B are enumeratable. Prove that there are  $A', B' \subseteq \mathbb{N}$  such that  $X = A' \setminus B'$  and A' and B' are enumeratable.