Lecture 8: Partial Orderings

Definition

Let R be a binary relation on a set X.
We say that R is an equivalence relation iff the following is true.

(reflexivity) for any $x \in X$, xRx

$(x_1, y_1) R (x_1, y_1) \iff x_1 + y_1 = x_1 + y_1$

(symmetry) for any $x, y \in X$, xRy iff yRx

$(x_1, y_1) R (x_2, y_2) \iff x_1 + y_2 = x_2 + y_1$

(transitivity) For any $x, y, z \in X$, if xRy and yRz, then xRz.

$(x_1, y_1) R (x_2, y_2)$ $(x_2, y_2) R (x_3, y_3)$

$x_1 + y_2 = x_2 + y_1$ $x_2 + y_3 = x_3 + y_2$

$x_1 + y_2 - y_1 = x_2$ $x_1 + y_2 - y_1 + y_3 = x_3 + y_2$

Exerise

Show that the relation R on V_{st}.

$(x_1, y_1) R (x_2, y_2)$ iff $x_1 + y_2 = x_2 + y_1$ is an equivalence relation.

$x_2 + y_1 = x_1 + y_2$

$(x_2, y_2) R (x_1, y_1)$

Definition A binary relation R on S is a partial ordering if it satisfies the following constraints

(reflexivity)

for any x, xRx

(transitivity)

$$\forall x, y, z \quad xRy, \; yRz \Rightarrow xRz$$

(antisymmetry)

$$xRy \text{ and } yRx \Leftrightarrow x = y$$

We say that a partial ordering R is total (total ordering) iff for any $x, y \in S$ either xRy or yRx.

Let $S \subseteq \mathbb{R}$ and R be a relation on S s.t

xRy iff $x \leq y$.

- $x \leq x$ for any $x \in S$
- $x \leq y$ $y \leq z$ then $x \leq z$
- $x \leq y$ $y \leq x$, then $x = y$

Exercise

- Give 3 examples of partial orderings.
- Which one of them are total?

Let $S = 2^{[n]}$

$x R y \iff x \leq y$

$\{1\} \not\leq \{2\}$ $\{2\} \not\leq \{1\}$