Lecture 3: Functions and Relations

Definition

union

1. $A \cup B$ is the set of all x s.t. $x \in A$ or $x \in B$

intersection

2. $A \cap B$ is the set of all x s.t. $x \in A$ and $x \in B$.
3. 2^A is the set of subsets of A.

Powers set $P(A)$

Exercise - How many elements in $2^{\{1,2,3\}}$?

- How many elements in $\{1,2,3\} \cap \{2,3\}$?
- How many elements in $\{1,2\} \cup \{2,3\}$?

Definition

Function $f: X \to Y$ is a unique assignement of elements of Y to the elements of X.

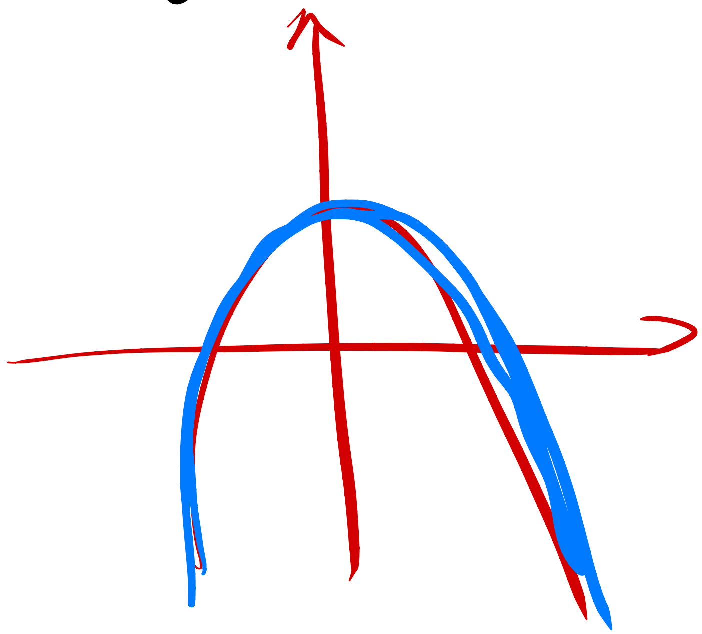
Exercise

Give two examples of functions.

Give an example of correspondence that is not a function

- $f: \{1,2\} \to \{2,3\}$ s.t. $f(x) = \begin{cases} 2 & \text{if } x=1 \\ 3 & \text{if } x=2 \end{cases}$
- $f: \mathbb{R} \to \mathbb{R}$ s.t. $f(x) = x$

How can we define functions formally using the notion of a set?



$\{(x, x^2) : x \in \mathbb{R}\}$

$G \subseteq X \times Y$ is a graph of a function iff

- For any $x \in X$ there is $y \in Y$ s.t. $(x, y) \in G$
- For any $x \in X$, if $(x, y_1) \in G$ and $(x, y_2) \in G$, then $y_1 = y_2$

Universal quantifier

For any real number x, x^2 is greater than 0.

$$\forall x \in \mathbb{R}\ x^2 > 0$$

Existential quantifier

For any real number x, there is a real number y such that $y^2 = x$.

$$\forall x \in \mathbb{R}\ \exists y \in \mathbb{R}\ y^2 = x$$

$$\forall x \in \mathbb{R}_{\geq 0}$$

Definition

Let X and Y be some set, and $f: X \to Y$ be a function from X to Y

Im f is the set of all possible values of f.

Exercise

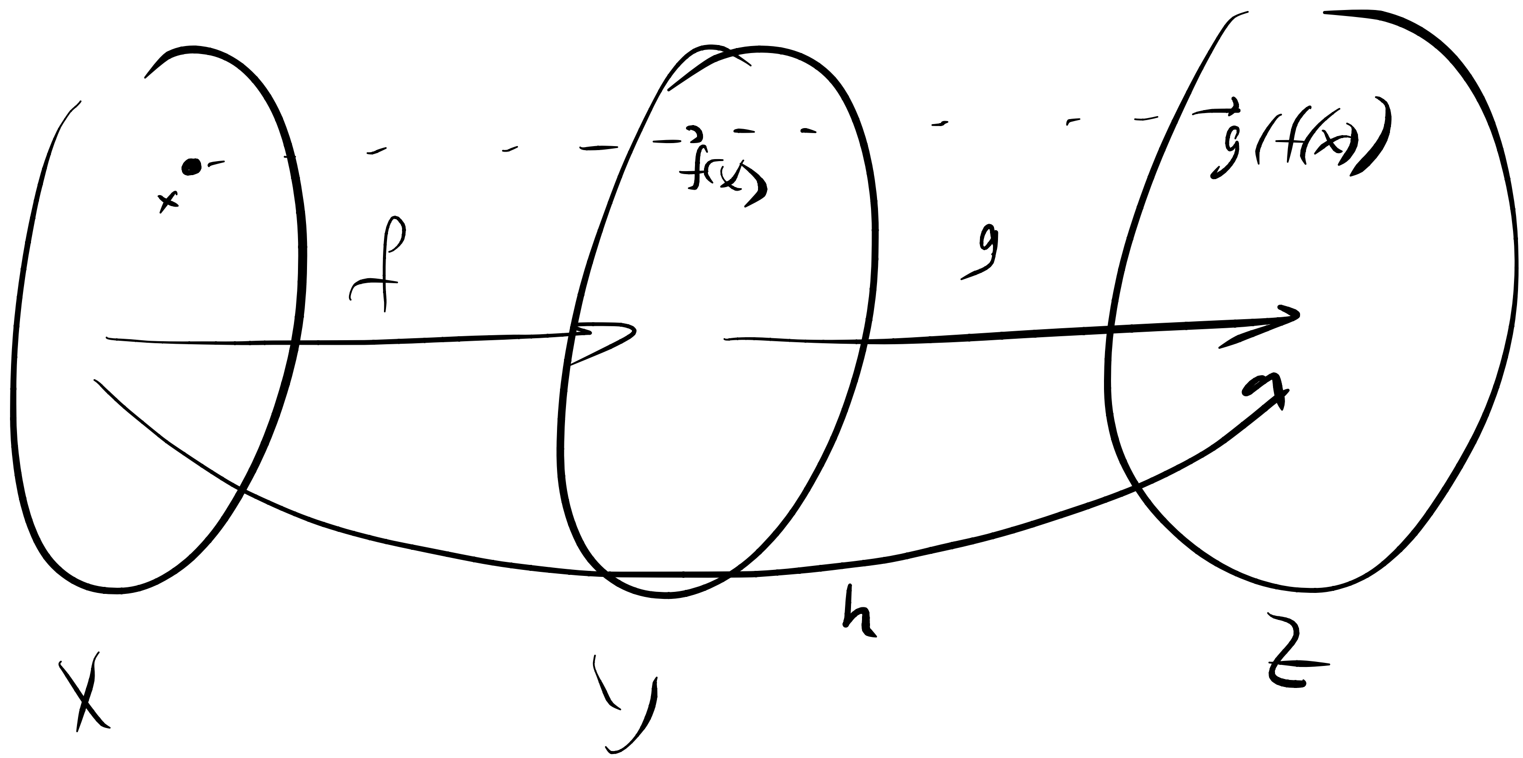
Give two formulas defining Im f

$\mathrm{Im}\, f = \{ f(x) : x \in X \}$

$\mathrm{Im}\, f \subseteq Y$, s.t. $\forall y \in \mathrm{Im}\, f \ \exists x \in X \ f(x) = y$

Definition

Let $f: X \to Y$, $g: Y \to Z$ be some functions. Then $h = g \circ f$ is the composition of g and f iff $h(x) = g(f(x))$ for any $x \in X$



Exercise

Define functions f and g s.t. $f \circ g \neq g \circ f$

$(f, g : X \to X)$

Let $f(x) = x^2$ $\quad X = \mathbb{R}$

$g(x) = -x$

$f(g(x)) = x^2$

$g(f(x)) = -x^2$