# Lecture 21

## Definition

Let $S$ be a signature and $\mathcal{M}$ be a str. with this signature.

Consider $\varphi$ a prop. formula in $S$ s.t. $v_1 \ldots v_k$ are the only free variables in $\varphi$

Let $a_1 \ldots a_k$ be elements from $\mathcal{M}$

Then $\mathcal{M} \models \varphi[a_1 \ldots a_k]$ iff

$\mathcal{M} \models \varphi[s]$, where $s$ is an asign.

s.t. $s(v_i) = a_i$ for $i \in [k]$.

We say that $R \subseteq M^k$ is repr. in $\mathcal{M}$ iff there is a prop. formula in $S$ s.t.

$$\{(a_1, \ldots, a_k) : \mathcal{M} \models \varphi[a_1, \ldots, a_k]\} = R$$

(where $v_1 \ldots v_k$ are the only free vars. of $\varphi$)

Let $S = (=; <)$ and $\mathcal{M} = (\mathbb{Z}; =, <)$

$R \subseteq \mathbb{Z}^2$ s.t. $(x, y) \in R$ iff $y = x + 1$

Claim

$R$ is repres. in $\mathcal{M}$.

Indeed, consider $\neg(\exists z \; x < z \wedge z < y)$

Let $S = (=;+, y=x^2)$ $\mathcal{M} = (\mathbb{R}; =; +, y=x^2)$. Show that $R \subseteq \mathbb{R}^3$ s.t. $(x,y,z) \in R$ iff $xy = z$ is representable in $\mathcal{M}$.

You know that $(x+y)^2 = x^2 + 2xy + y^2$.

So $x^2 + 2z + y^2 = (x+y)^2$ iff $z = xy$

Consider $\varphi$ equal to $((x^2 + z) + z) + y^2 = (x+y)^2$

Let $S = (x|y)$ and $\mathcal{M} = (\mathbb{N}; x|y)$. Show that $R \subseteq \mathbb{N}$ s.t. $R = \{1\}$ is representable in $\mathcal{M}$.

Show that $R \subseteq \mathbb{N}$ s.t. $x \in R$ iff $x$ is prime is representable in $\mathcal{M}$.

Show that $R \subseteq \mathbb{N}^2$ s.t. $(x,y) \in R$ iff $x = y$ is repr. in $\mathcal{M}$.

Consider $\varphi$ equal to $x|y \wedge y|x$

Consider $\psi$ equal to $\forall y \; (y|x) \Rightarrow \varphi$