Name: ____________________

Pid: ____________________

1. (a) (10 points) Let ϕ, ψ, and χ be propositional formulas on Ω. Show that $\big(\phi \vee (\psi \wedge \chi)\big)\big|_\rho = \big((\phi \vee \psi) \wedge (\phi \vee \chi)\big)\big|_\rho$ for any assignment ρ to the variables Ω.

   (b) (10 points) Let $\psi_{1,1}, \ldots, \psi_{1,n}, \psi_{2,1}, \ldots, \psi_{2,m}$ be propositional formulas on Ω. Let $\phi_1 = \bigwedge_{i=1}^n \psi_{1,i}$ and $\phi_2 = \bigwedge_{j=1}^m \psi_{2,j}$.

   Show that $\big(\phi_1 \vee \phi_2\big)\big|_\rho = \big(\bigwedge_{i=1}^n \bigwedge_{j=1}^m (\psi_{1,i} \vee \psi_{2,j})\big)\big|_\rho$ for any assignment ρ to the variables Ω.

   (c) (10 points) Let Ω be a set of variables. We say that a propositional formula is a literal if the formula is equal to x or $\neg x$ for $x \in \Omega$.

   We say that a propositional formula on Ω is in conjunctive normal form if it is equal to $\bigwedge_{i=1}^n \bigvee_{j=1}^{m_i} \psi_{i,j}$, where $\psi_{i,j}$ is a literal.

   Let ϕ be a propositional formula on Ω. Show using structural induction that there is a propositional formula ψ on Ω in conjunctive normal form such that $\psi\big|_\rho = \phi\big|_\rho$ for any assignment ρ to Ω.