Name: _____

Pid: _____

1. (10 points) Let us define a union of more than two sets as follows. Let $A_1, \ldots, A_n$ be some sets. Then

   - $\bigcup_{i=1}^{1} A_i = A_1$ and
   - $\bigcup_{i=1}^{k+1} A_i = \left(\bigcup_{i=1}^{k} A_i\right) \cup A_{k+1}$.

   Show that $\bigcup_{i=1}^{n} [i] = [n]$ for all integers $n > 0$.

2. (10 points) Let us define an intersection of more than two sets as follows. Let $A_1, \ldots, A_n$ be some sets. Then

   - $\bigcap_{i=1}^{1} A_i = A_1$ and
   - $\bigcap_{i=1}^{k+1} A_i = \left(\bigcap_{i=1}^{k} A_i\right) \cap A_{k+1}$.

   Show that $\bigcap_{i=1}^{n} \{x \in \mathbb{N} \ : \ i \leq x \leq n\} = \{n\}$ for all integers $n > 0$.