Name: ____________________

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1. Let us cosider two planes defined by the equations $2(x-3)+(y-4)+(z-1)=0$ and $(x-1)+3(y-2)+2(z-7)=0$.

   Find the angle between these planes.

   Solution: Note that the angle between the planes is the same as the angle between the normales. Note that $n_1 = \langle 2,1,1\rangle$ and $n_2 = \langle 1,3,2\rangle$.

   Hence, the answer is equal to $\arccos \frac{n_1 \cdot n_2}{|n_1|\cdot|n_2|} = \arccos \frac{2\cdot 1+1\cdot 3+1\cdot 2}{\sqrt{6}\cdot\sqrt{14}} = \arccos \frac{7}{\sqrt{84}}$.

2. Check if the lines defined by the equations $\frac{x-2}{3} = \frac{y-1}{2} = \frac{z}{4}$ and $\frac{x-1}{3} = \frac{y-2}{2} = \frac{z-1}{2}$ are intersecting.

Solution: Let us write the parametric forms of the lines. Let $r_0 = \langle 2, 1, 0\rangle$, $r_1 = \langle 1, 2, 1\rangle$, $v_0 = \langle 3, 2, 4\rangle$, and $v_1 = \langle 3, 2, 2\rangle$. Hence, the parametric forms are $r_0 + t_0 v_0$ and $r_1 + t_1 v_1$, respectively.

In order to check whether the lines are intersecting we need to check if there are t_0 and t_1 such that $r_0 + t_0 v_0 = r_1 + t_1 v_1$ i.e.

$$\begin{cases} 2 + 3t_0 = 1 + 3t_1 \\ 1 + 2t_0 = 2 + 2t_1 \\ 4t_0 = 1 + 2t_1. \end{cases}$$

This implies that $t_0 = \frac{1+2t_1}{4}$ and, as a result, $1 + \frac{1+2t_1}{2} = 2 + 2t_1$. Hence, $1 + 2t_1 = 2 + 4t_1$ i.e. $t_1 = -1/2$ and $t_0 = 0$. But this contradicts to the first equality.

So these lines are not intersecting.

3. Find symmetric equations for the line of intersection of the planes defined by the equations $(x-1)+2(y-1)+(z-4)=0$ and $2(x-2)-y+(z-5)=0$.

Solution: First of all let us find a point in intersection ℓ of these planes. Both of these planes intersecting the plane xy, we find a point from the xy plane in the line ℓ. To do this we fix $z=0$ and solve the system of equations

$$\begin{cases} (x-1)+2(y-1)-4=0 \\ 2(x-2)-y-5=0 \end{cases} .$$

The solution of this system is $x=5$ and $y=1$.

We now need to find the direction of the line. To do this we just neet to compute the vector product of normales (since the direction is perpendicular to both of them):

$$\langle 1,2,1\rangle \times \langle 2,-1,1\rangle = (2+1)i-(1-2)j+(-1-4)k = \langle 3,1,-5\rangle.$$

As a result, the answer is

$$\frac{x-5}{3} = y-1 = -\frac{z}{5}.$$