Name: ____________________

Pid: ____________________

1. (50 points) Check all the correct statements (in this question only the answers will be graded).
   - ○ $\gcd(24, 18) = 6$.
   - ○ The function $f : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \to \mathbb{R}$ such that $f(x) = \arctan x$ is a bijection.
   - ○ The cardinality of the set $F(X, [3]) = (4^n)^3$, where $X = F([4], [n])$.
   - ○ The cardinality of the set $I([3], [n]) = n(n-1)(n-2)$.
   - ○ $\binom{10}{2} = 90$.

2. (a) (5 points) Let n, a, and b be some integers. Show that if two numbers a and b have the same reminders when divided by n, then $a - b$ is divisible by n.

(b) (5 points) Prove that for every integers $a_1, \ldots, a_n$ there are $k > 0$ and $\ell \geq 0$ such that $k + \ell \leq n$ and $\sum_{i=k}^{k+\ell} a_i$ is divisible by n.

3. (10 points) We say that sets A_1, A_2, and A_3 are pairwise disjoint iff $A_i \cap A_j = \emptyset$ for every $i \neq j \in [3]$. Construct a bijection from $\{0, 1, 2, 3\}^n$ to $\{(A, B, C) \mid A, B, C \subseteq [n] \text{ and } A, B, C \text{ are pairwise disjoint}\}$

4. (10 points) How many numbers from [999] are not divisible neither by 3, nor by 5, nor by 7.

5. (10 points) Let m be some integer. Show that product of m consecutive integers is divisible by $m!$.